AP Calculus - 2012-09-11

2.3/22

$$\lim_{y\to+\infty} \frac{2-y}{\sqrt{7+6y^2}} = \lim_{y\to+\infty} \frac{y\left(\frac{2}{y}-1\right)}{\sqrt{y^2}\sqrt{\frac{7}{y^2}+6}}$$

$$= \lim_{y\to+\infty} \frac{\cancel{y}\left(\frac{2}{y}-1\right)}{\cancel{y}\sqrt{\frac{7}{y^2}+6}} = \frac{-1}{\sqrt{6}}$$

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2.3/26)

$$\lim_{x \to -\infty} \frac{5-2x^3}{x^2+1} = \lim_{x \to -\infty} \frac{x^3\left(\frac{5}{x^3}-2\right)}{x^2\left(1+\frac{1}{x^2}\right)}$$

$$= \lim_{x \to -\infty} \left(\frac{x^2}{x^2}\right) \cdot \frac{x\left(\frac{5}{x^3}-2\right)}{\left(1+\frac{1}{x^2}\right)} = +\infty$$

$$\lim_{x\to-\infty} \frac{\sqrt{3x^2+x}}{x-8} = \lim_{x\to-\infty} \frac{\sqrt{x^2}\sqrt{3+\frac{1}{x}}}{x\left(1-\frac{8}{x}\right)}$$

$$\sqrt{x^2} = \begin{cases} x & \text{if } x\geq 0 \\ -x & \text{if } x<0 \end{cases} = |x|$$

$$= \lim_{x\to-\infty} \frac{|x|\sqrt{3+\frac{1}{x}}}{x\left(1-\frac{8}{x}\right)} = \lim_{x\to-\infty} \frac{-x\sqrt{3+\frac{1}{x}}}{x\left(1-\frac{8}{x}\right)}$$

$$= \frac{-\sqrt{3}}{1} = -\sqrt{3}$$

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2.4)

$$\lim_{x \to 2} f(x) = L$$

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Preamble. $f(x)$ is defined on an open interval of x-values.
$x=a$ is a value in that open interval.
Except... it's OK if $f(a)$ is not defined.

$\lim_{x \to a} f(x) = L$ if, iff and only if

small Greek epsilon

given any number $\epsilon > 0$,

small Greek delta

we can find a number $\delta > 0$,

such that $|f(x) - L| < \epsilon$ for every x with
$0 < |x-a| < \delta$

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2.4/1 a) Find largest open interval, centered on $x=0$,

so that $\forall x$ in that open interval,

for all (for every)

$f(x)$ is within $.01$ units of $y=2$

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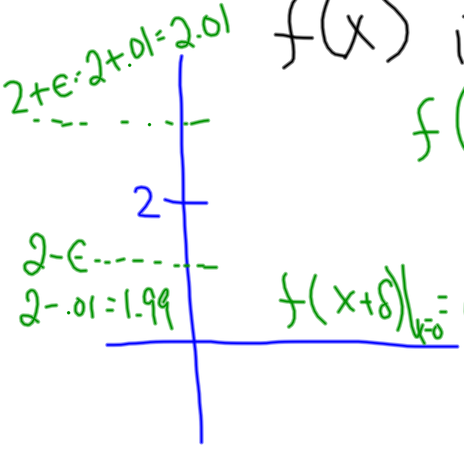
$f(x)=x+2$

$f(0)=2$



$f(\text{"}x+\delta\text{"}) = \text{"}f(x)+\epsilon\text{"}$

$0 \qquad 0 \qquad .01$

$f(x+\delta)\big|_{x=0} = (0+\delta)+2 = 2+\delta \leq 2+.01 = 2.01$

$$\begin{aligned} 2+\delta &\leq 2.01 \\ -2 \quad & \quad\; -2 \end{aligned} \Rightarrow \boxed{\delta \leq .01}$$